

A

1. Which of the following phenomena could be explained by classical physics and did *not* require a quantum hypothesis in order to make theory agree with experiment?

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|---|-------------------------|
| (a) Blackbody spectra | (e) Atomic line spectra |
| <input type="checkbox"/> (b) Diffraction of light | (f) (b) and (e) |
| (c) Low-temperature heat capacity in perfect crystals | (g) (a), (b), and (d) |
| (d) The photoelectric effect | (h) (c) and (d) |

2. If a normalized wave packet Ψ is given as $\Psi(x, y, z, t) = \sum_{n=1}^{\infty} c_n \psi_n(x, y, z) e^{-iE_n t / \hbar}$, what is the probability that an experiment will cause the system to collapse to the specific stationary state j ?

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| (a) Quantum mechanics does not allow you to know this probability | (e) c_j |
| (b) One | <input type="checkbox"/> (f) $ c_j ^2$ |
| (c) $\langle \psi_j H \psi_j \rangle$ | (g) (b) and (d) |
| (d) c_j^* | (h) Only Schrödinger's cat knows |

3. Which of the following statements about the de Broglie wavelength λ are *false*?

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| (a) λ decreases as mass increases if velocity is constant | (e) A particle that has zero velocity has an infinite de Broglie wavelength |
| (b) $\lambda = h / p$ | (f) All of the above |
| (c) λ increases as momentum increases | (g) (a), (b) and (e) |
| (d) λ is a constant, like Planck's constant | <input type="checkbox"/> (h) (c) and (d) |

4. Which of the following statements about a well behaved wave function is *true*?

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| (a) It must be continuous | (e) Its square modulus has units of probability density |
| (b) It may take on complex values | (f) It must be an eigenfunction of the momentum operator |
| (c) It must be quadratically integrable | (g) (d) and (f) |
| (d) It must be equal to its complex conjugate | <input type="checkbox"/> (h) (a), (b), (c), and (e) |

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5. Which of the below equations will be true for any pair of orthonormal functions f and g that are eigenfunctions of the Hamiltonian H ?

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| (a) $fg = 0$ | (e) $\langle f g \rangle = 0$ |
| (b) $\langle f ^2 \rangle \langle g ^2 \rangle = 1$ | (f) (a) and (c) |
| (c) $f^*g - g^*f = 0$ | (g) (b), (d) and (e) |
| (d) $\langle f H g \rangle = 0$ | (h) All of the above |

6. Which of the below expectation values are or may be non-zero?

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| (a) $\langle \sin x x \cos x \rangle$ | (e) $\langle f g \rangle - \langle g f \rangle$ |
| (b) $\langle \sin^2 x x \cos^2 x \rangle$ | (f) $\langle \mu_{mn} \rangle$ for a forbidden transition |
| (c) $\langle f [A, B] g \rangle$ where A and B commute | (g) (a), (d) and (e) |
| (d) $\langle \Psi H \Psi \rangle$ where Ψ is a stationary state | (h) (b), (c), and (f) |

7. Which of the following did Bohr assume in order to derive a model consistent with the photoemission spectra of one-electron atoms?

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| (a) The electron is a delocalized wave | (e) The ionization potential is equal to the work function |
| (b) The angular momentum of the electron is quantized | (f) (a) and (b) |
| (c) The one-electron atom is like a particle in a box | (g) (b) and (d) |
| (d) The Coulomb potential is quantized | (h) None of the above |

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8. Which of the following statements are *false* about the free particle?

- (a) Its Schrödinger equation is $\left(-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} - E\right)\Psi(x) = 0$ (e) Valid wave functions include $\Psi(x) = Ae^{ikx} + Be^{-ikx}$ where $k = \frac{\sqrt{2mE}}{\hbar}$
- (b) It may be regarded as having a wave function that is the superposition of a left-moving particle and a right-moving particle (f) Valid wave functions include $\Psi(x) = N \cos kx$ where N is a normalization constant and k is defined in (e) above
- (c) Its energy levels are all non-negative (g) All of the above
- (d) Its energy levels are not quantized (h) None of the above

9. Given a particle of mass m in a box of length L having the wave function

$$\Psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right), \text{ what is the energy of the level corresponding to } n = 4?$$

- (a) Since this wave function is not an eigenfunction of the Hamiltonian the question cannot be answered (e) $\frac{8\pi^2 \hbar^2}{mL^2}$
- (b) 16 times the energy of the ground state (f) (c) and (d)
- (c) $\langle \Psi | p_x^2 | \Psi \rangle$ (g) (b) and (e)
- (d) $8\hbar^2 / mL^2$ (h) None of the above

10. On which of the below functions does the parity operator Π act in the fashion $\Pi[f(x)] = (-1)f(x)$?

- (a) x (e) Any eigenfunction of the Hamiltonian
- (b) x^2 (f) (b) and (d)
- (c) e^{ix} (g) (a), (b), and (d)
- (d) $\cos x$ (h) (b), (d), and (e)

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Short-answer (20 points)

Prove that, given a pair of normalized but *not* orthogonal functions ψ_1 and ψ_2 , the function $\psi_3 = \psi_2 - S\psi_1$ is orthogonal to ψ_1 if S is the overlap integral of ψ_1 and ψ_2 . Is ψ_3 normalized? (Use the back of the page if necessary).

We evaluate

$$\begin{aligned}\langle \psi_1 | \psi_3 \rangle &= \langle \psi_1 | \psi_2 - S\psi_1 \rangle \\ &= \langle \psi_1 | \psi_2 \rangle - S \langle \psi_1 | \psi_1 \rangle \\ &= S - S \cdot 1 \\ &= 0\end{aligned}$$

which proves the orthogonality (we use normalization of ψ_1 in going from line 2 to 3).

With respect to normalization, we evaluate

$$\begin{aligned}\langle \psi_3 | \psi_3 \rangle &= \langle \psi_2 - S\psi_1 | \psi_2 - S\psi_1 \rangle \\ &= \langle \psi_2 | \psi_2 \rangle - 2S \langle \psi_1 | \psi_2 \rangle + S^2 \langle \psi_1 | \psi_1 \rangle \\ &= 1 - 2S \cdot S + S^2 \\ &= 1 - S^2\end{aligned}$$

so ψ_3 is only normalized for the boring case of $S = 0$ (which means that ψ_1 and ψ_2 were orthogonal to begin with since no change to ψ_2 is required to generate ψ_3).

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