B

1. Which of the following phenomena could not be explained by classical physics and required a quantum hypothesis in order to make theory agree with experiment?

(a) Blackbody spectra
(b) Diffraction of light
(c) Low-temperature heat capacity in perfect crystals
(d) The photoelectric effect
(e) The motion of a pendulum
(f) (a), (c), and (d)
(g) (a), (b), and (d)
(h) None of the above

2. Which of the following did Bohr *not* assume in order to derive a model consistent with the photoemission spectra of one-electron atoms?

(a) The electron is a delocalized wave
(b) The angular momentum of the electron is quantized
(c) The one-electron atom is like a particle in a box
(d) The Coulomb potential is quantized
(e) The ionization potential is equal to the work function
(f) (a) and (b)
(g) (a), (c), (d), and (e)
(h) None of the above

3. Which of the following statements about the de Broglie wavelength λ are *true*?

(a) $\lambda = -h / p$
(b) λ increases as momentum increases
(c) λ decreases as mass increases if velocity is constant
(d) A particle that has zero velocity has an infinite de Broglie wavelength
(e) λ is a constant, like Planck's constant
(f) All of the above
(g) (a), (b) and (e)
(h) (c) and (d)

4. Which of the below equations can be *false* for an arbitrary pair of orthonormal functions *f* and *g*?

(a) $< |f|^2 >< |g|^2 > < 1$
(b) $< f \mid H \mid g > = 0$
(c) $< f \mid g > \neq < f \mid g >^*$
(d) $fg = 0$
(e) $f^*g - g^*f = 0$
(f) (a) and (c)
(g) (b), (d) and (e)
(h) All of the above

NAME: ____

5. Which of the following statements about a well behaved wave function is *false*?

(a) It must be nowhere equal to zero

(b) It may not take on complex values

(c) It must be equal to its complex conjugate

(d) It must be quadratically integrable

(e) It must be an eigenfunction of the momentum operator

(f) Its square modulus has units of probability density

(g) (d) and (f)

(h) (a), (b), (c), and (e)

6. Which of the below expectation values are zero?

(a) $< \sin x \mid x \mid \cos x >$

(b) $< \sin^2 x \mid x \mid \cos^2 x >$

(c) $< \Psi \mid H \mid \Psi >$ where Ψ is a stationary state

(d) $< f \mid [A,B] \mid g >$ where A and B commute

(e) $<f \mid g> - <g \mid f>$

(f) $<\mu_{mn}>$ for a forbidden transition

(g) (b), (d) and (f)

(h) (b), (c), and (f)

7. If a normalized wave packet Ψ is given as $\Psi(x,y,z,t) = \sum_{n=1}^{\infty} c_n \psi_n(x,y,z) e^{-iE_n t/\hbar}$, what is the probability that an experiment will cause the system to collapse to the specific stationary state j?

(a) Quantum mechanics does not allow you to know this probability

(b) $c_j^* c_j$

(c) One

(d) $|c_j|^2$

(e) c_j

(f) c_j^*

(g) (b) and (d)

(h) Only Schrödinger's cat knows

NAME: ____

8. Which of the following statements are true about the free particle?

(a) Its time-independent Schrödinger equation is $\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} - E\right)\Psi(x) = 0$

(b) It may be regarded as having a wave function that is the superposition of a left-moving particle and a right-moving particle

(c) Its energy levels are all non-negative

(d) Its energy levels are not quantized

(e) Valid wave functions include $\Psi(x) = Ae^{ikx} + Be^{-ikx}$ where $k = \frac{\sqrt{2mE}}{\hbar}$

(f) Valid wave functions include $\Psi(x) = N\cos kx$ where N is a normalization constant and k is defined in (e) above

(g) All of the above

(h) None of the above

9. Given a particle of mass m in a box of length L having the wave function $\Psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$, what is the energy of the level corresponding to n = 4?

(a) Since this wave function is not an eigenfunction of the Hamiltonian the question cannot be answered

(b) 16 times the energy of the ground state

(c) $< \Psi \mid p_x^2 \mid \Psi >$

(d) $8\hbar^2 / mL^2$

(e) $\frac{16\pi^2\hbar^2}{mL^2}$

(f) (c) and (d)

(g) (b) and (e)

(h) None of the above

10. On which of the below functions does the parity operator Π act in the fashion Π[$f(x)$] = (1)$f(x)$?

(a) x

(b) x^{-1}

(c) e^{ix}

(d) $\cos x$

(e) Any eigenfunction of the Hamiltonian

(f) (b) and (d)

(g) (a), (b), and (d)

(h) (b), (d), and (e)

NAME: ______________________________

Short-answer (20 points)

Prove that, given a pair of normalized but *not* orthogonal functions ψ_1 and ψ_2, the function $\psi_3 = \psi_2 - S\psi_1$ is orthogonal to ψ_1 if S is the overlap integral of ψ_1 and ψ_2. Is ψ_3 normalized? (Use the back of the page if necessary).

We evaluate

$$\begin{aligned}\langle\psi_1|\psi_3\rangle &= \langle\psi_1|\psi_2 - S\psi_1\rangle \\ &= \langle\psi_1|\psi_2\rangle - S\langle\psi_1|\psi_1\rangle \\ &= S - S\bullet 1 \\ &= 0\end{aligned}$$

which proves the orthogonality (we use normalization of ψ_1 in going from line 2 to 3).

With respect to normalization, we evaluate

$$\begin{aligned}\langle\psi_3|\psi_3\rangle &= \langle\psi_2 - S\psi_1|\psi_2 - S\psi_1\rangle \\ &= \langle\psi_2|\psi_2\rangle - 2S\langle\psi_1|\psi_2\rangle + S^2\langle\psi_1|\psi_1\rangle \\ &= 1 - 2S\bullet S + S^2 \\ &= 1 - S^2\end{aligned}$$

so ψ_3 is only normalized for the boring case of $S = 0$ (which means that ψ_1 and ψ_2 were orthogonal to begin with since no change to ψ_2 is required to generate ψ_3).

NAME: ____