B

1. For the diatomic molecule CD, where C has atomic mass 12 and D has atomic mass 2, what is the reduced mass?

(a) 7 / 12

(b) 12 / 7

(c) 1 / 7

(d) 2 / 7

(e) 10

(f) It depends on the vibrational state

(g) Cannot be determined from information given

(h) None of the above

2. Which of the following statements is/are *true* for a given set of QMHO wave functions corresponding to the same harmonic potential $V$?

(a) The ground state energy is zero, i.e., the bottom of the potential

(b) The number of nodes is equal to $n$+1, where $n$ is the energy level

(c) $\langle T\rangle_n = \langle V\rangle_n = \langle E\rangle_n$

(d) The wave functions have zero amplitude beyond the classical turning points

(e) The wave functions are eigenfunctions of the parity operator

(f) The selection rule for spectroscopic transitions is $n \rightarrow n \pm 2$

(g) (c), (e), and (f)

(h) All of the above

3. Which of the following statements about angular momentum operators and their eigenvalues and eigenfunctions is/are *true*?

(a) $L_+ = -(L_-)^*$

(b) $\langle L^2\rangle = \langle L_z\rangle^2$ whenever $m_l = l$

(c) For each value of $l$ there are $2l$ possible values of $m_l$

(d) $L_+ Y_{l,l} = 0$

(e) $[L_x, L_y] = 2\hbar L_z$

(f) $\langle Y_{l,0}|T|Y_{l,0}\rangle > \langle Y_{l',0}|T|Y_{l',0}\rangle$ if $l < l'$

(g) (b) and (f)

(h) All of the above

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4. What is the eigenvalue of $L_z$ for $\Psi$ if the eigenvalue of $L^2$ for $\Psi$ is $25\hbar^2$ and the eigenvalue of $\left(L_x^2 + L_y^2\right)$ for $\Psi$ is $16\hbar^2$?

| | | | |
|---|---|---|---|
| (a) | The Heisenberg uncertainty principle dictates that $\Psi$ cannot be an eigenfunction for $L_z$ | (e) | $\boxed{\pm 3\hbar}$ |
| (b) | $16\hbar^2$ | (f) | 0 |
| (c) | $4i\hbar^2$ | (g) | $\pi$ |
| (d) | $\pm 4\hbar$ | (h) | None of the above |

5. For a diatomic rigid rotator having reduced mass 3 and bond length 2 a.u., which of the following statements is/are false?

| | | | |
|---|---|---|---|
| (a) | The ground-state energy is zero | (e) | The moment of intertia is 12 a.u. |
| (b) | The energy separation between the first and second excited states is ( 1 / 6 ) a.u. | (f) | (a) and (c) |
| (c) | The rotational constant $B$ is ( 1 / 24 ) a.u. | (g) | (b) and (e) |
| (d) | Transition from the ground state to the state $J = 3$ is forbidden | (h) | $\boxed{\text{None of the above}}$ |

6. For a spin-free hydrogenic wave function, which of the below relationships between quantum numbers is/are always true?

| | | | |
|---|---|---|---|
| (a) | $n = l > m_l$ | (e) | $\boxed{n > l \geq m_l}$ |
| (b) | $n = l + m_l$ | (f) | (b) and (c) |
| (c) | $n > l + m_l$ | (g) | (b) and (e) |
| (d) | $n > l > m_l$ | (h) | None of the above |

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7. Which of the below statements about electron spin is/are true?

(a) The spin quantum number comes from including gravity in the electronic Schrödinger equation

(b) Spin couples with orbital angular momentum according to $\mathbf{J} = \mathbf{L} - \mathbf{S}$

(c) For a single electron, the only eigenvalues of $S_z$ are $\pm(1/2)\hbar$

(d) Stern and Gerlach discovered electron spin by studying the magnetic moments of Au atoms

(e) Spin-orbit coupling is proportional to the 8th power of the atomic number

(f) (a) and (c)

(g) (a), (c), and (d)

(h) All of the above

8. An electron of spin $\alpha$ is in a 5f orbital. Which of the below sets of quantum numbers ($n$, $l$, $m_l$, $m_s$) might describe such an electron?

(a) (5, 4, 4, 4)

(b) (4, 3, 2, 1/2)

(c) (5, 3, 0, 1/2)

(d) (4, 5, 0, 7/2)

(e) (4, 4, 5, 1/2)

(f) (c) and (e)

(g) (b), (c), (d) and (e)

(h) None of the above

9. What is the ground-state ionization potential for a one-electron atom having atomic number $Z$?

(a) $Z^2$ a.u.

(b) The negative of the energy of the electron in the 1s orbital

(c) $(1/2)Z^2$ a.u.

(d) $2Z^2$ a.u.

(e) 1 a.u.

(f) The energy required to infinitely separate the nucleus and electron

(g) (b), and (d)

(h) (b), (c), and (f)

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10. Which of the following wave functions has the greatest degeneracy?

(a) Particle in a box, level $n = 8$

(b) Rigid rotator, $l = 4$

(c) Quantum mechanical harmonic oscillator, level $n = 25$

(d) Spin-free hydrogenic wave function, $n = 6$, $l = 1$

$\boxed{\text{(e) Spin-free hydrogenic wave function, } n = 4}$

(f) Relativistic free electron at rest

(g) (b) and (e) have the same degeneracy which is greater than all of the others

(h) (a) through (f) are all singly degenerate

**Short answer.** Show that by proper choice of $a$, the function $e^{-ar^2}$ is an eigenfunction of the operator

$$\left[\frac{d^2}{dr^2} - qr^2\right]$$

where $q$ is a constant. What is the name of the general class of functions represented by $e^{-ar^2}$? How many nodes does this function have over $r$?

To show that $e^{-ar^2}$ is an eigenfunction with proper choice of $a$ we require

$$\left[\frac{d^2}{dr^2} - qr^2\right]e^{-ar^2} = ze^{-ar^2}$$

Evaluating the l.h.s. we have

$$\begin{aligned}\left[\frac{d^2}{dr^2} - qr^2\right]e^{-ar^2} &= -2ae^{-ar^2} + 4a^2r^2e^{-ar^2} - qr^2e^{-ar^2} \\ &= -\left[2a + \left(q - 4a^2\right)r^2\right]e^{-ar^2}\end{aligned}$$

and for the prefactor on the r.h.s. to be a constant (so as to satisfy the eigenvalue condition) it must be true that $a$ is $\sqrt{q}/2$ in which case the eigenvalue will be $-2a$, which is simply $-\sqrt{q}$.

The general class of functions here are "gaussian" functions. A gaussian has no nodes over $r$.

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