Some Potentially Useful Mathematical Formulae

Trigonometric Relations

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha-\beta)+\sin(\alpha+\beta)\right]$$

$$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha$$

$$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

Some Operators

$x \equiv$ multiply by x

$\mathbf{r} \equiv$ multiply by $\mathbf{r}$

$$p_x \equiv -i\hbar\frac{d}{dx}$$

$$H = T + V$$

$$\boldsymbol{\mu} \equiv e\mathbf{r}$$

$$\mathbf{L} \equiv \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ -i\hbar\dfrac{\partial}{\partial x} & -i\hbar\dfrac{\partial}{\partial y} & -i\hbar\dfrac{\partial}{\partial z} \end{vmatrix}$$

$$L_+ = L_x + iL_y \quad \text{and} \quad L_- = L_x - iL_y$$

Integrals

$$\int_0^L \sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi x}{L}\right)dx = \frac{L}{2}\delta_{mn}$$

$$\int x\cos(ax)dx = \frac{x}{a}\sin ax + \frac{\cos ax}{a^2}$$

$$\int x^2\cos(ax)dx = \frac{2x\cos ax}{a^2} + \frac{a^2x^2-2}{a^3}\sin ax$$

$$\int_0^\infty r^n e^{-2r}dr = \frac{n!}{2^{n+1}}$$

Complex Relations

$$\sqrt{-1} = i = -\frac{1}{i}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Spherical Polar Volume Element

$$r^2 dr\sin\theta d\theta d\phi$$

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