

Some Potentially Useful Mathematical Formulae

Integrals

$$\int_0^{\infty} e^{-ax^2} dx = \frac{1}{2} \left(\frac{\pi}{a} \right)^{1/2}$$

$$\int_a^b u dv = uv \Big|_a^b - \int_a^b v du$$

$$\int_0^{\infty} x e^{-ax^2} dx = \frac{1}{2a}$$

$$\int_0^{\infty} x^2 e^{-ax^2} dx = \frac{1}{4a} \left(\frac{\pi}{a} \right)^{1/2}$$

$$\int_0^{\infty} x^3 e^{-ax^2} dx = \frac{1}{a^2}$$

$$\int_0^{\infty} x^n e^{-ax^2} dx = \begin{cases} \frac{(n-1)!}{2a^{(n+1)/2}}, & n \text{ odd} \\ \frac{1 \cdot 3 \cdot 5 \cdots (n-1)}{2(2a)^{n/2}} \left(\frac{\pi}{a} \right)^{1/2}, & n \text{ even} \end{cases}$$

$$\int_0^{\pi} \sin \theta d\theta \int_0^{2\pi} d\phi = 4\pi$$

Other

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} ae + bf \\ ce + df \end{bmatrix}$$

$$\sqrt{-1} = i = -\frac{1}{i}$$

$$e^{i\theta} = \cos \theta + i \sin \theta$$

Some Operators

$x \equiv$ multiply by x

$$T = -\frac{1}{2r^2} \left(\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} - L^2 \right) \text{ [spherical polars]}$$

$$p_x \equiv -i\hbar \frac{d}{dx}$$

$$H = T + V$$

volume element in spherical polar
coordinates: $r^2 \sin \theta dr d\theta d\phi$

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